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ON CONVECTION IN STELLAR ATMOSPHERES FAR FROM LOCAL THERMODYNAMIC EQUILIBRIUM

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Abstract. Non-thermal effects generated by sub-photospheric convection are considered. It is shown that convective cells are destroyed by shocks generated when convective velocities reach the speed of sound. Terminologically this process is given the name of 'sonic-boom-interrupted convection'. An estimate is made on the dependence of convective velocities on stellar parameters. It is suggested that the process being investigated could explain why some stars do not belong to any branch of the theoretical Hertzsprung–Russell diagram.

1. Introduction

An attempt is made to explain the simultaneous occurrence of emission lines, strong blue continuum and violent macroscopic movements in the atmospheres of some giant stars and other objects. Scarce evidence is available on the origin of high-speed motions. In the present approach they are thought to be connected to large-scale sub-photospheric convection, assuming that the violent sub-photospheric convection breaks into shock waves, thereby generating non-thermal processes in the atmosphere. We shall first consider the conditions under which shock waves are directly generated by convection.

Electrodynamic processes can produce non-thermal effects if a hydrodynamically driven dynamo-mechanism is supposed to generate a strong sub-photospheric magnetic field. In this case, it is assumed that (a) the appearance of non-thermal phenomena is not only indirectly connected to strong sub-photospheric convection but the sudden dissipation of field energy can be brought about directly by hydrodynamic processes, and (b) non-thermal phenomena appear as a consequence of the interaction between the magnetic field and the violent convection. Biermann (1948) suggested a special way in which shock waves and non-thermal effects of mechanical origin can be generated. In his model the mean radial fluctuation of convective velocities turns into shock waves. However, this theory, in which the velocities are supposed to be low compared to the speed of sound, is unable to account for high-energy flares since the velocity fluctuations could never transport thousand times the value of the total energy radiated from a quiet star. This model does not seem to work for stellar atmospheres which are far from being in thermodynamic equilibrium.

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2. A New Aspect of the Generation of Shock Waves

In the earlier conceptions the top of the outer convective zone is taken to be a layer in which the density and temperature are the same as those of the convective cells. Upon reaching the photosphere, the material of the convective elements mixes with that of the environment which is in radiative equilibrium. It can be shown (Grandpierre, 1977) that there are given physical conditions under which the convective transfer will cease in a manner different from the way described above. Let us now examine in detail the physical background which causes the convection to cease.

According to the Schwarzschild criteria there exists an interval, given as $r_1 < r < r_2$ (see Figure 1), which is in convective instability. In this interval the ascending convective cells are accelerated at an increasing rate by a growing buoyant force. The rate of acceleration starts to decrease in the interval $r_2 < r < r_3$, and at r_3 the buoyant force goes to zero. Inert convective elements which overshoot this point are gradually stopped by the negative buoyant force acting in the interval $r_3 < r < r_4$. Dissipative effects are neglected in the above consideration.

Convective cells can be accelerated to the speed of sound before reaching the interval of the negative buoyant force if the actual temperature gradient 'sufficiently' differs from the adiabatic gradient (see Figure 2). Although some investigators (e.g., Hoyle and Schwarzschild, 1955) have already pointed out that in some stars convective velocities could reach the speed of sound, no in-

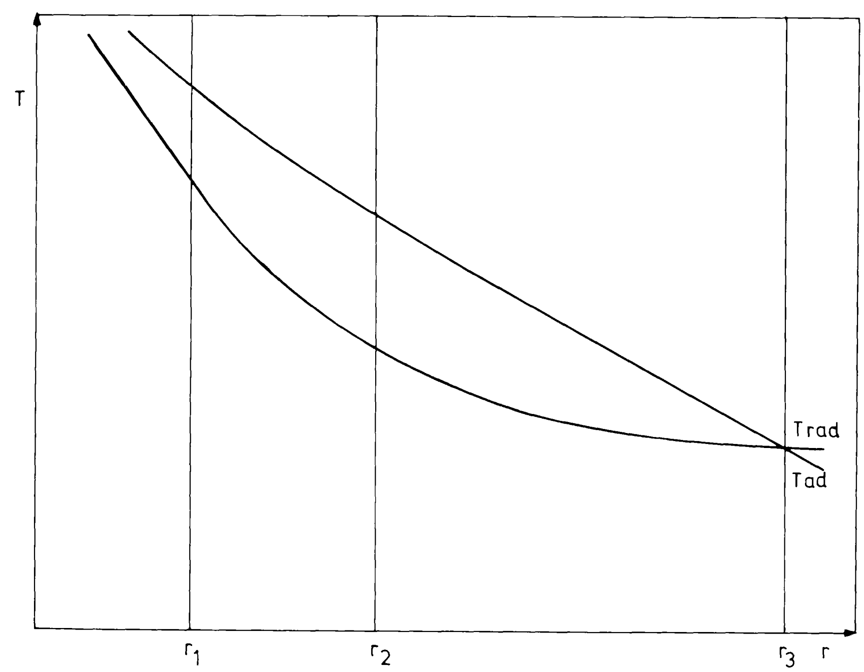

Fig. 1. Temperature distribution when convection ceases with zero velocity; r is the distance from the centre of the star.

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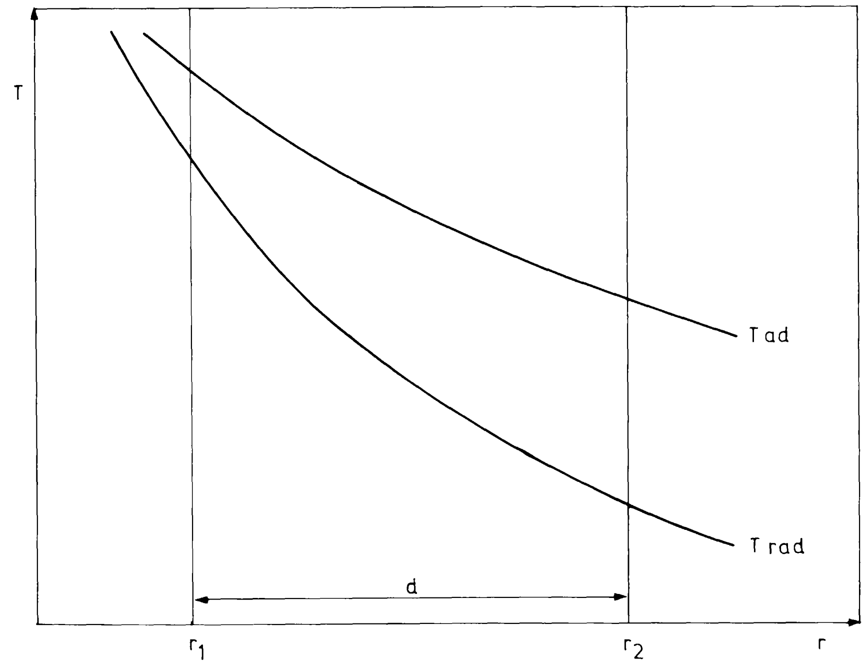

Fig. 2. Temperature distribution in the case of sonic-boom-interrupted convection.

ferences have been made concerning the effect of the onset of shock waves on convection when velocities reach the threshold of a sonic boom. It is now assumed that the convective cells are destroyed by the shocks so that the material of the moving elements is promptly mixed with that of their environment. Let us now see this process in detail.

The ascending convective elements which start to move at subsonic velocity are more and more accelerated by the buoyant forces while the density, frictional forces and the speed of sound decrease outwards. However, as the velocities approach the threshold Mach number $M = 1$, there is a sudden increase in the frictional forces (Figure 3) and convective cells conserving their kinetic energy run into the sharp front of high friction. This sonic boom destroys the inner organization of the convective cells and the shocks generate surfaces with abrupt changing thermodynamic parameters. This means that the density and temperature perturbations are unbalanced; thus the local thermodynamic equilibrium (LTE) is being destroyed. The convection ceases to be a macroscopically organized large-scale flow. The material of convective cells mixes with that of the environment. The surplus internal energy and the kinetic energy transported by convection turn into shock energy heating the atmosphere.

Let us now estimate the order of the temperature gradient required by the convective velocities to reach the speed of sound over a pathlength l, assumed to be identical with the mixing length. The buoyant force is given by the expression

$$F = \Delta\rho\, gV = \rho Va\,, \tag{1}$$

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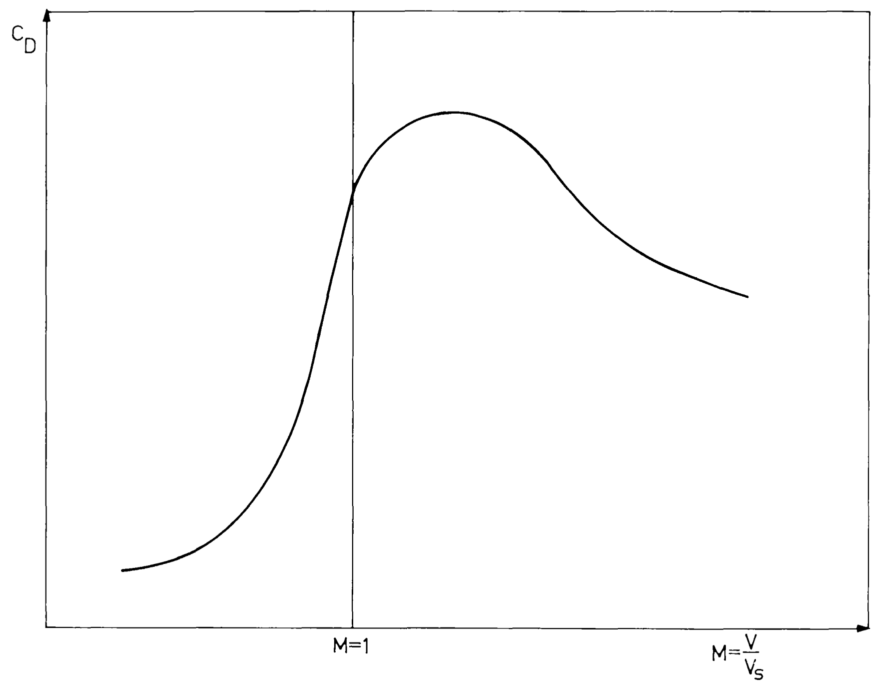

Fig. 3. The rapid growth of the non-dimensional drag coefficient C_D in the vicinity of Mach number $M = 1$ (see Cox and Crabtree, 1965).

where ρ is the density of the convective cells, $\Delta\rho$ the density difference between the convective cells and their environment, g the gravitational acceleration, and V and a are, respectively, the volume and acceleration of the convective cells, where

$$a = \frac{\Delta\rho}{\rho} g . \tag{2}$$

In the absence of ionization, we have

$$\frac{\Delta\rho}{\rho} = \frac{\Delta T}{T} , \tag{3}$$

where T is the temperature of the convective cells and ΔT is the temperature difference between the convective cells and their environment. Assuming uniform acceleration, we can write

$$v^2 = 2 \frac{\Delta T}{T} gl , \tag{4}$$

where v is the velocity of the convective elements. The temperature difference, expressed in terms of the actual superadiabatic gradient ($\mathrm{SAG_{act}}$) has the form

$$\Delta T = l \left| \left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{act}} - \left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}} \right| \equiv l \, \mathrm{SAG_{act}} . \tag{5}$$

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If we substitute (5) for (4) and put $v^2 = v_s^2 = \gamma \mathcal{R} T$, where v_s is the speed of sound, γ is the ratio of the specific heats and $\mathcal{R}$ is the universal gas constant, the critical value of the superadiabatic gradient at which the convective velocities can reach the speed of sound is obtained as

$$\mathrm{SAG}_{\mathrm{crit}} = \gamma \mathcal{R} T^2/(2gl^2) . \tag{6}$$

Thus, the condition of sonic-boom-interrupted convection can be expressed as

$$\mathrm{SAG}_{\mathrm{act}} > \mathrm{SAG}_{\mathrm{crit}} . \tag{7}$$

$(\mathrm{d}T/\mathrm{d}r)_{\mathrm{ad}}$ can be evaluated from spectroscopic observations by use of the well-known relation

$$\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}} = -\frac{g\mu}{\mathcal{R}}\frac{\gamma - 1}{\gamma} , \tag{8}$$

where μ is the mean molecular weight. This relation also permits $\mathrm{SAG}_{\mathrm{act}}$ to be calculated. If we assume some realistic value for the mixing length, an estimate can be made of $\mathrm{SAG}_{\mathrm{crit}}$. As we have no observational evidence on $(\mathrm{d}T/\mathrm{d}r)_{\mathrm{act}}$ except for the Sun, relation (7) is tested for solar granulae.

If, for example, we take $l = 1000$ km (the mean diameter of the granulae) and $T = 7.5 \times 10^3$ K with $g = 3 \times 10^4$ cm s^{-2}, we obtain the critical superadiabatic gradient of the solar granulae as $\mathrm{SAG}_{\mathrm{crit}} = 10^{-5}$ K cm^{-1}. Observational data give $\mathrm{SAG}_{\mathrm{act}} = 2 \times 10^{-4}$ K cm^{-1}. Now, if we take into account that Equation (6) underestimates the value of $\mathrm{SAG}_{\mathrm{crit}}$, it seems that the result is consistent with the fact that solar granulae move at a velocity of 1–2 km s^{-1} in the atmosphere having a local speed of sound of 7 km s^{-1}.

3. Theoretical and Observational Arguments

Convective elements can be accelerated to the speed of sound if they remain optically thick when reaching the photosphere, since optically thin convective elements impart their surplus internal energy to their environment and mix with the photospheric matter (see, e.g., Henyey, 1965). Schwarzschild (1975) has shown that there are some red giants in which convective elements could remain optically thick even as they approach the photosphere.

Moreover, sound waves produced by turbulent convection do not essentially dissipate the kinetic energy of the convective elements. This fact is confirmed by Lighthill's conclusion (1967) that sound of a given frequency can only be generated by eddies that are too small in relation to the acoustic wavelength to radiate efficiently.

Observations show that there are some stars with atmospheres in which convection actually reaches the speed of sound. Wright's observational data (1955) listed in Table I show that in the tabulated stars there are some high macroturbulent velocities which can be treated in terms of sonic or supersonic

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TABLE I

Star	Spectral class	Macroturbulent velocity (km s^{-1})
ρ Leonis	B1 Ib	8
ε Canis Majoris	B2 II	7
55 Cygni	B3 Ia	6
γ Geminorum	A1 IV	13
α Cygni	A2 Ia	22
δ Canis Majoris	F8 Ia	9
η Aquilae	F6–G3	8

velocities. At this point, it seems necessary to note that in the atmosphere of a typically hot O star the speed of sound is below $20\,\text{km s}^{-1}$, so that it could be thought that convective velocities might exceed the local speed of sound in the atmosphere of some stars. However, as we have already shown, sonic boom prevents convective velocities in stellar atmospheres from becoming supersonic, and no convective velocities can exceed the speed of sound in this case. The high velocities given in Table I need further analysis by taking into account the deviations of the atmospheric conditions from both hydrostatic and thermodynamic equilibria.

4. Estimate of Convective Velocities

Let us now formulate the dependence of convective velocities on stellar parameters.

The energy flux transported by convection can be expressed (Reddish, 1974) by

$$F = \frac{Q\rho v}{\mu} = 2\times 10^{8}\,\frac{\rho v\,\Delta T}{\mu}\,, \tag{9}$$

where Q is the internal.energy. Assuming that convective energy transport is dominant in the convective zone, we can write

$$F = L/(4\pi R^{2})\,, \tag{10}$$

where L is the luminosity and R the radius of the star. By making use the equation of state, $p_c = (k/\mu H)\rho_c T_c$, where k is Boltzmann's constant, H is the mass of the hydrogen atom, $p_{c_1} T_c$ and ρ_c are, respectively, the central pressure, temperature and density of the star, Equation (9) can be rewritten in the form

$$F = 2\times 10^{8}\,\frac{H}{k}\,p_c\,\frac{\Delta T}{T_c}\,\frac{\rho}{\rho_c}\,v\,. \tag{11}$$

By use of the integral theorems of equilibrium (Chandrasekhar, 1957), the central

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pressure can be estimated as

$$p_c = \alpha \frac{3}{8\pi} \frac{GM^2}{R^4}, \tag{12}$$

where $\alpha \geqslant 1$, G is the gravitational constant and M and R are the mass and radius of the star, respectively. By Equations (9), (11) and (12) we get

$$v = \frac{\alpha}{3 \times 10^8} \frac{k}{H} \frac{LR^2}{GM^2} \frac{T_c}{T} \frac{\rho_c}{\rho}. \tag{13}$$

For massive stars having convective cores, the mass–luminosity relation is given by the Strömgren theorem as

$$L \propto M^3. \tag{14}$$

Thus, Equations (10), (13) and (14) can be used to estimate the velocity in terms of T_c/T and ρ_c/ρ as

$$v \propto MR^2 \frac{T_c}{T} \frac{\rho_c}{\rho}. \tag{15}$$

This dependence means that on giant stars with high central temperatures the convection in the atmosphere can be interrupted by a sonic boom since the velocity of the convection can grow to a very high value due to its proportional increase to that of M, R^2 and T_c. It has to be noted here that high values of these parameters can be simultaneously present, and for this reason there must be high convective velocities in some giant stars where convection is expected to cease as a consequence of sonic boom. In the case of red giants the characteristic dimensions of the convective zone and of convective cells are large and the convective elements can remain optically thick (see Schwarzschild, 1975). This estimate of convective velocities agrees with Wright's observational results, showing high macroturbulent velocities for some giant stars (Table I).

5. Conclusions

In the computation of a stellar model there are parameters which cannot be derived from observations, so that we must estimate their value by use of assumptions. In fact, as the conditions in stellar atmospheres are affected by sub-photospheric convection, any estimate of the missing data is difficult.

If we want to consider in the stellar atmospheres the cessation of convective transfer because of sonic boom, the considerations concerning the surface conditions of the stellar model must be revised. We have to take into account that the inner energy and the kinetic energy of convective cells, which are high and comparable with each other at precisely the speed of sound, are transported to the photosphere by shock waves in the case of sonic-boom-interrupted convection. For this reason the surface temperature of such stars is expected to

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be higher than that of stars in which convection ceases with zero velocity. The structure of the former – let us call them SB stars – are substantially different from those of normal stars. The existence of these SB stars is suggested by the discrepancies in the fit of the theoretical H–R diagram to the observational diagram. The flare stars below the Main sequence present problems in that they are not interpretable in terms of the contractive stellar model (Hayashi, 1966). There are quantitative inconsistencies of the theory with observations of Population I 5 $M_\odot$ stars (where $M_\odot$ is the mass of the Sun) which are attributed to the uncertainties in the convective flow parameters (Iben, 1967).

The statistical occurrence of SB stars as inferred from these discrepancies seems to be low. On the other hand, it is known that the H–R diagram is computed with the use of some important free parameters and with arbitrary adjustments of the theory to observations so that some of the stars classified as normal are actually SB stars. The fitting procedure is not always sound: for example, the efficiency of convection, which is in very close connection with convective velocities, is chosen as a free parameter to fit the blue and red supergiant branches of the cluster h and χ Persei to the observational H–R diagram (Hayashi, 1962). On the other hand, just some supergiants could be qualified as SB stars.

The strong blue continuum, the emission lines and the violent macroscopic motions can be interpreted by sonic-boom-interrupted convective transfer in stellar atmospheres. A study of T-Tauri stars and flare stars (Grandpierre, 1981) suggests that the convection in their atmospheres ceases with sonic boom. The interpretation of stellar spectra in terms of the proposed model requires the study to be continued in more detail.

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