ON THE ONSET OF THERMAL METASTABILITIES IN THE SOLAR CORE

ATTILA GRANDPIERRE[1,*] and GÁBOR ÁGOSTON[2]

[1]*Konkoly Observatory of the Hungarian Academy of Sciences, H-1525 Budapest, Hungary; E-mail: grandp@konkoly.hu;* [2]*Gustin Software, H-1159 Budapest, Késmárk u. 1, Hungary; E-mail: gustin@freemail.hu*

(Received 4 December 2003; accepted 7 July 2004)

Abstract. For infinitesimal, homologous perturbations, stability analysis has found the solar radiative interior thermally stable. It is considered for the first time here whether stability is preserved when finite amplitude nonhomologous perturbations are present. We argue that local heated regions may develop in the solar core due to magnetic instabilities. Simple numerical estimations are derived for the timescales of the decay of these events and, when heated bubbles are generated that rise towards the surface, of their rising motion. These estimations suggest that the solar core is in a metastable state. For more detailed analysis, we developed a numerical code to solve the differential equation system. Our calculations determined the conditions of metastability and the evolution of timescales. We obtained two principal results. One of them shows that small amplitude heating events (with energy surplus $Q_o < 10^{26}$ ergs) contribute to subtle but long-lifetime heat waves and give the solar interior a persistently oscillating character. Interestingly, the slow decay of heat waves may make their accumulation possible and so their overlapping may contribute to the development of an intermittent, individual, local process of bubble generation, which may also be generated directly by stronger ($Q_o > 10^{26}$ ergs) heating events. Our second principal result is that for heated regions with $\Delta T/T \geq 10^{-4}$ and radius 10^5–10^6cm, the generated bubbles may travel distances larger than their linear size. We point out to some possible observable consequences of the obtained results.

Keywords: solar core dynamics, metastability, sporadic mixing, solar activity

1. Introduction

Pantha rei, everything flows, tells the ancient Greek saying (Heracleitus). Modern science seems to agree with this notion. Cosmic systems change on many timescales simultaneously. At the same time, linear stability analysis of Schwarzschild and Harm (1973), Rosenbluth and Bahcall (1973) and Unno (1975) indicated that the solar core is stable to internal, infinitesimal, radial and nonradial, homologous perturbations and so it is a general view that there is no dynamism present in the solar core. This view was strengthened by the solar model calculations of Gilliland (1985) that indicated that the solar core may be stable for finite amplitude homologous perturbations of element abundances. Recently, Paterno et al. (1997) reconsidered nonradial thermal instabilities in the solar core for internal, infinitesimal, homologous, i.e., shellular perturbations. They have found that the solar core is stable against such perturbations. Remarkably, on the basis of their results we may conjecture that the solar core is close to instability for finite amplitude nonradial perturbations. This circumstance is due to the fact that the heating timescale they obtained – for homologous perturbations produced by nuclear heating – $\tau_{\text{growth}} \approx 4 \times 10^6$ years is only slightly higher than the cooling one, arising from radiative diffusion $\tau_{\text{decay}} \approx 7 \times 10^5$ years (see Table 2; at the solar centre). In this paper, we show that in the solar radiative interior heat waves and sporadically hot bubbles are generated that travel significant distances towards the surface. The actual presence of heat waves and energetic, sporadic, bubble-like perturbations in the solar radiative interior necessarily generates deviations from the perfect symmetry of the quiescent solar model, and the solar core may be unstable for such a type of perturbations. We found that the generation of heated regions present a new, yet not considered type of instability that lends a certain dynamism to the solar core which may have a fundamental significance in the origin of solar activity.

At present, in solar (and, in general, stellar) physics, the radiative interior is frequently considered to be in quasistatic equilibrium changing only on the equilibrium nuclear timescale 10^{11} years. Bahcall (1989) characterized the situation with the following sentence: "The Sun's interior is believed to be in a quiescent state and therefore the relevant physics is simple."

At the same time, it is well known that instabilites arise from the action of a magnetic field in a rotating plasma. Such instabilities can well generate thermal inhomogeneities in localized regions. Unfortunately, at present the detailed description of such nonlinear phenomenon is an extremely difficult problem. Actually, the intensity of the magnetic field in the solar radiative zone is not known excepts for its upper limit 3×10^7 G (Couvidat et al., 2002). It is important to note that the presence of a magnetic field is in fact highly plausible on the basis of its slow rotation. The magnetic instabilities constitute the most efficient mechanism for transferring the angular momentum from the core to the surface. Therefore, we may regard the presence of such a magnetorotational instability mechanism plausible.

We know that the solar core is in a plasma state. Plasmas are extremely complicated physical systems fundamentally different from classic neutral gases, especially when there is a magnetic field present. In this paper we argue that the real solar core is rich in dynamical phenomena in a wide range of temporal and spatial scales. Within such conditions, plasma processes will also set up and they will lead to the development of significant nonlinear phenomena. In the solar core the gaseous pressure ($\approx 10^{17}$ dyne) is much larger than the magnetic pressure since the average toroidal field has a helioseismic upper limit of 3×10^7 G (Couvidat et al., 2002). Nevertheless, the criterion for neglecting magnetic effects in the treatment of a problem in gas dynamics is that the Lundquist parameter $L_u = (4\pi)^{1/2}\sigma B l_c / c^2 \rho_m^{1/2}$ (measuring the ratio of the magnetic diffusion time to the Alfven travel time, where σ is the electric conductivity in e.s.u., B is the strength of the magnetic field in Gauss, l_c is a characteristic length of the plasma in centimeter, and ρ_m is the mass density in gcm^{-3}, c is the speed of light), is much less than unity, $L_u \ll 1$ (Peratt, 1992, 19). Now for the solar core $\sigma \approx 10^{17}$ e.s.u.,

$B \approx 10^{-3}$ to 3×10^7 G, $l_c \approx 10^{10}$ cm, $\rho_m \approx 10^2$ gcm^{-3}, and so $L_u \approx 4 \times 10^4$ to 10^{15}, i.e., $L_u \gg 1$. Therefore, plasma effects may play a dominant role in the dynamism of the solar core. We note that even when $L_u \ll 1$, hydrodynamic movements (or self-organization of stochastic radiation field (see Li and Zhang, 1996) may amplify the magnetic fields to values $L_u \gg 1$ later on.

It is well known that the solar core is changing on a wide variety of timescales (Turck-Chieze, 2001) due to magnetic instabilities (Spruit, 2002). The necessity of a dynamic solar core model is indicated by many independent theoretical and observational arguments (Grandpierre, 1990, 1996), and a trend towards the dynamical representation of the stars is noticed by Turck-Chieze (2001).

In a highly conductive plasma such complex, nonlinear effects necessarily lead to electric currents. Since unstable nonlinear effects tend to grow exponentially, even subtle changes may grow to significant amplitudes. "The collective plasma processes are associated in particular with various plasma instabilities. As a rule the development of instability is accompanied by an increase in the electric field strength, which can attain large values. Consequently, even in the absence of intense external fields, relatively strong fields can still occur spontaneously in a plasma due to the growth of instability" (Tsytovich, 1970). Plasma microinstabilities are localized, usually high frequency phenomena that cannot be described in MHD but only in the kinetic models. Such plasma microinstabilities may set up due to deviations from the isotropic Maxwell-Boltzmann distribution, or to inhomogeneities in the electric or magnetic fields, density or temperature fluctuations, collisions, etc. The energy of the magnetic field shows an accumulation process that is released at irregular time intervals in the form of flares (Contopoulos, 2003). If significant parts of the energies of the solar core may be localized into small regions, these regions may be heated to relatively high temperatures and hot bubbles may form.

In this paper one of our main aims is to explore the physics of these hot bubbles. Therefore, we present here a short description of the energy localizing processes of the different energy forms in the solar core. We know that it is almost impossible to generate stable magnetic configurations in earthly fusion reactors. Simple magnetohydrodynamical calculations (e.g., Tayler, 1973; Spruit, 2002) indicated the general presence of MHD instabilities in the solar radiative zone. Plasma instabilities have a local and nonlinear character. The dissipation of the energies of moving plasma in localized regions can lead to pinches and condensed states (Peratt, 1996). Recently, Chang et al. (2003) demonstrated that the sporadic and localized interactions of magnetic coherent structures arising from plasma resonances generate the "complexity" in space plasmas. For typical MHD turbulence, the arising coherent structures are generally flux tubes. What we find really important is that there are some actually possible mechanisms that may lead to a localization of energy in a highly concentrated form and this energy concentration is a natural consequence of the nonlinearity of the related equations.

It was Li and Zhang (1996) who solved the reduced nonlinear two-fluid equations and they had shown that the self-organization of the stochastic thermal radiation field is in a close relation with the self-generated magnetic field. They obtained that a magnetic field stronger than 400 MG may be generated at the center of the Sun showing that the self-organizing behavior of the stochastic radiation field does occur. The growth time scale of the self-generated magnetic field is about 10^{12} sec.

Besides these theoretical arguments underpinning the sporadic localization of energy liberation in the plasma of the solar core, we also have some observational support indicating the presence of heated regions and flare-like phenomena in the solar radiative interior. One may recognize the signs of flare-like events in the deep solar core on the basis of the observations of Toutain and Kosovichev (2001) and Chaplin et al. (2003). They found an anomalous event at late March 1998 supplying additional energy to solar activity and low-1 solar p-modes. This event raised the velocity power (V^2, which is directly proportional to the total energy of a mode) by 22% above the zero change level; the predicted value for this epoch in the cycle, however, is of the order of $\approx -5\%$. By our best knowledge, similar energy enhancements of p-modes are observed until now only in relation to flares (Haber et al., 1988; Kosovichev and Zharkova, 1998). Chaplin et al. (2003) noted that the increase of energy supply is coincident in time with the southern hemisphere onset of cycle 23, with a major emission of particles and the appearance of major surface activity on this hemisphere. Remarkably, Benevolenskaya (1999) had been shown, that the transition from cycle 22 to cycle 23 clustered in the very same fixed longitudinal regions. Recognizing that such activity enhancements are usually related to active regions with especially high flare activity, one may assume that the increase of energy supply is related to a certain localised event somewhere in the solar interior. Because this event is energetic and localized, one may apply the term "flare-like event." Such flare-like events, if occurring in the radiative zone, may be related to bubble generation.

Considering if the origin of this flare-like events may occur in the deep radiative zone, we call the attention to the recent results of Bai (2003). Bai (2002) paid attention to the fact that solar flares from the southern hemisphere during cycle 23 are found to be concentrated in a pair of hot spots rotating with a synodic period of 28.2 days. Moreover, Bai (2003) has found that the hot spots of this double hot-spot system are separated by about 180° longitude. Many hot-spot systems last for more than one solar cycle, and therefore the mechanism(s) generating them must be independent of toroidal magnetic fluxes. Since the toroidal fields are found around the top of the radiative zone, the mechanism(s) generating the hot spots must act below the zone containing the toroidal flux tubes. Taking into account the facts that hot-spot systems set up frequently in a 180° longitudinal separation, and that they have an anomalous rotation rate from 25 to 29 days, a range surpassing the range of rotational periods observed both on the surface and in the convective zone in the latitude zone extending from $-35°$ to $35°$, one may seem plausible to find the origin of hot spots deep in the solar core. Actually, helioseismic measurements allow such anomalously rotating layers or regions if their spatial scales are less than 100 km. Therefore, the localization of the source of hot spots suggest the presence of strongly heated localised regions deep in the solar core, a circumstance favourable for the generation of hot bubbles. It seems plausible to think that the source of hot spots may be related to the flare-like events which produce the increased energy supply for solar activity and p-modes in March 1998 (Chaplin et al., 2003).

These theoretical and observational results indicate that the main energy forms of the solar core tend to form sporadic localized heated regions, making the solar core a dynamic, active system (Grandpierre, 1990, 1996; Turck-Chieze, 2001).

The consideration of bubble-like perturbations creates a new situation in comparison to the shellular case. Bubble-like perturbations may couple hydrodynamic instabilities to thermal perturbations. Moreover, finite-amplitude perturbations may be relevant and generate significant motions also in cases when linear stability analysis indicates no movements. Therefore, it is interesting to follow such finite amplitude bubble-like perturbations individually by numerical computations. In this way, we can determine the parameters of the arising hydrodynamic movements, including the distance a heated bubble may travel, and this parameter may be an important indicator of the dynamism of the solar core.

2. Basic Equations

We can start with the conservation equations of momentum and energy, together with the equation of state. We have for the k-component of momentum per unit volume

$$\partial(\rho v^k)/\partial t + \sum_i \partial/\partial x^i(\rho v^k - P^{ik}) = \rho X^k, \tag{1}$$

where X^k is the k-component of the total force acting per unit mass, and P^{ik} is the total stress tensor. The conservation of energy tells that

$$dU/dt + p/\rho div v = \epsilon_N - 1/\rho div(F_R + F_c) + 1/\rho \sum_{i,k} P^{ik}\partial v_k/\partial x^i, \tag{2}$$

where U is the total thermal energy, ϵ_N is the liberated nuclear energy per unit mass and time, F_R and F_c are the radiative and conductive fluxes. The equation of state is

$$p = (R_g/\mu)\rho T, \tag{3}$$

where μ is the dimensionless mean molecular weight, and R_g is the gas constant.

3. Basic Estimations for the Case of a Heated Bubble

To obtain a preliminary picture about the possibility whether heated bubbles may or may not travel a distance larger than their characteristic sizes, first we determined the relevant timescales of this process. This basic calculation is simple and provides a substantial insight into the physics of the solar core, namely, into its stability against such sporadic localized bubble-like motions. It is a favorable method also because it offers a fast and easy way to make a first explorational step into this kind of stability considerations.

We formulate the following scenario: a dissipation event heats a local parcel of matter in the solar interior at $r = 0.1 R_\odot$. We calculated how this initial perturbation generates a heated bubble (or heated region, in case of microinstability) which is already in pressure equilibrium with its surroundings. The difference between a heated region and a heated bubble is that the heated region does not rise upwards. A heated bubble is not in hydrostatic equilibrium with its surroundings. In the first approximation, the motion of a heated bubble is determined by the equality of the buoyant ($F_b = Vg\Delta\rho$) and frictional ($F_f = K/2v^2 S\rho$) forces, where S is the cross section of the bubble, ρ is the density of the bubble, V is its volume, K is the coefficient of turbulent viscosity, ρ_S and $\Delta\rho$ are the density of the surroundings and the density difference between the bubble and its surroundings, and g is the gravitational acceleration. Equating these forces, $v^2 K/2S/V = g\Delta\rho/\rho_s$. Now assuming pressure equilibrium between the bubble (referred with no index) and its surroundings (referred with index S), $\rho T = \rho_S T_S$, $\Delta\rho/\rho_S = (1 - T_S/T)$. Taking $K = 1$ (Öpik, 1958), we obtain for the bubble's velocity

$$v = (8/3Rg(1 - T_S/T))^{1/2}. \tag{4}$$

With typical values in the solar core $T_S/T < 8/9$, $g = 2 \times 10^5$ cm s², $R = 10^5 - 10^6$ cm, $v \approx 2$ to 7×10^5 cm s^{-1} (Gorbatsky, 1964).

Now we turn to the energy equation. The heated bubble is not in thermal equilibrium with its surroundings. Below $T \approx 10^8$ K the radiation energy and pressure may be neglected compared to the material energy and pressure. The radiation energy must, of course, not be neglected in the flux term. In a co-moving frame, without energy sources, when radiation is the most effective dissipative factor, the energy equation may be simplified to the form

$$\partial U/\partial t = -1/\rho div F_R = -1/\rho div[D_R grad E_R], \tag{5}$$

where $E_R = aT^4$ is the radiation energy density, and a is the radiation-density constant. Assuming that one can apply the diffusion approximation, one obtains the usual equation $D_R = 1/3cl_{\rm ph}$, where c is the speed of light, $l_{\rm ph} = 1/(\kappa\rho)$ is the mean free path of a photon, and κ is a mean absorption coefficient. In the case of spherical symmetry, the corresponding diffusive radiative flux is

$$F_R = -(4ac/3\kappa\rho)T^3\partial T/\partial r. \tag{6}$$

Now returning to the simplified energy equation (5), with $U = C_pT$, and integrating it to the whole volume of the bubble,

$$C_p(\partial T/\partial t)V = -(1/p)4\pi R^2 F_R. \tag{7}$$

From this equation the thermal adjustment time is estimated, in a linear approximation, writing for $\partial T/\partial t \approx -\Delta T/\tau_{\rm adj}$, and for $\partial T/\partial r \approx \Delta T/R$, following Kippenhahn and Weigert (1990) as:

$$\tau_{\rm adj} = \kappa\rho^2 C_p R^2/(16\sigma T^3), \tag{8}$$

where $\sigma = ac/4$ is the Stefann – Boltzmann constant ($\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}). With typical values ($\kappa = 2$ cm^2 g^{-1}, $\rho = 90$ g cm^{-3}, $C_p = 2.1 \times 10^8$ erg K^{-1} mole^{-1}, $T = 10^8$ K, $R = 10^6$ cm), $\tau_{\rm adj} = 3 \times 10^3$ sec, while for $T = 10^7$ K, $\tau_{\rm adj} = 4 \times 10^6$ sec.

In order to put this fundamental thermal time scale into the context we are interested in, we may define a rising time scale for the bubbles

$$\tau_{\rm rise} = l_T/v, \tag{9}$$

where l_T is the temperature scale height in the solar core $l_T \approx 1.5 \times 10^{10}$ cm at $r = R_\odot/10$. With $v = 1.5\times10^5$ to 1.5×10^6 cm s^{-1}, $\tau_{\rm rise} \approx 10^5$ to 10^4 s. This means that for (at least) moderate heating (when $T/T_S > 1.0001$) the bubble may move so fast that its thermal cooling is slower than the decrease of the temperature of its environment (for a moment, we ignore the cooling arising from volume expansion; more detailed results are given later on). In such a case, the bubble cannot adjust its temperature to its environment, and the formation of a bubble may lead to a certain kind of instability (i.e., to the self-maintaining movement of the bubble) even if we disregard from any internal energy source.

The time scale of cooling of the bubbles arising from adiabatic volume expansion may be calculated following Gorbatsky (1964). Starting from $Q = C_V mT = 2\pi R^3 p$,

$$(dQ/dt)_{\rm exp} = -pd(4/3\pi R^3)/dt, \tag{10}$$

or

$$2\pi p(3R^2)(dR/dt)_{\rm exp} + 2\pi R^3(dp/dt)_{\rm adiab} = -p4\pi R^2(dR/dt)_{\rm exp}, \tag{11}$$

$$(dR/dt)_{\rm exp} = -R/5(1/p(dp/dt)), \tag{12}$$

$$\tau_{\rm exp} = -(1/5(1/p(dp/dr))v)^{-1} = 5H_p/v, \tag{13}$$

and so for the solar core around $r = 0.1.R_{\odot}$, $H_p = |1/p(dp/dr)|^{-1} \approx 7.3 \times 10^9$ cm is the pressure scale height at $r = 0.1R_{\odot}$, for $R = 10^5 - 10^6$ cm and at least moderate heating, with $v \approx 10^3 - 10^6$ cm, $\tau_{\rm exp}$ is usually in the range of 3×10^7 to 10^4 s.

From these estimations we can recognise the remarkable situation that all the three relevant time scales determining the behavior of the bubbles are comparable to each other. Therefore, it is important to consider the case by more detailed numerical calculations and determine if there exist suitable conditions for triggering instability.

Our basic estimation formulates the condition of surfacing. The bubble may reach the solar subsurface regions when it runs a distance $\Delta r = \tau_{\rm adj} \times v$ which is much longer than the characteristic size of the bubble.

$$((K\rho\rho_S C_p R^2)/(16\sigma T^3))(8/3Rg(l - T_S/T))^{1/2} \gg R, \tag{14}$$

or, in more suitable form, if $T/T_S > 1.3$,

$$R_{\rm crit} \gg 1.1 \times 10^{-3}(1/g)^{1/3}(1/(\kappa\rho\rho_S c_p))^{2/3}T^2. \tag{15}$$

Numerically, $R_{\rm crit} \gg 10^3$ cm.

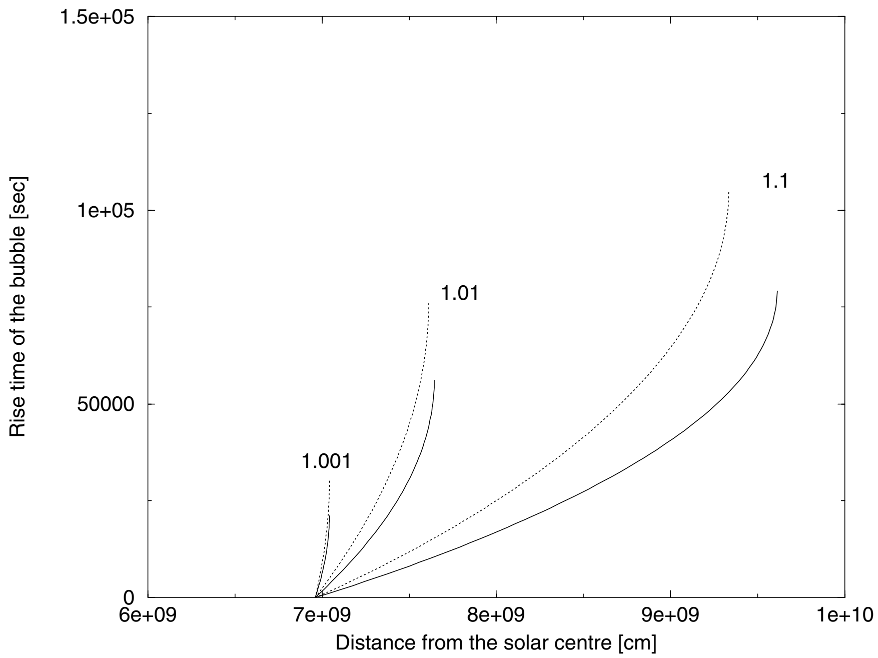

Figure 1. The travel of bubbles with different relative initial temperature surplus, for $R_0 = 10^5$ cm. $T_V/T_S = 1.001, 1.01, 1.1$. Dotted lines indicate the effect of deformation (modeled by $K = 2$) on the travel of bubbles.

We used the opacity figure of Rogers and Igliesas (1998), Figure 1, and approximated it as $\kappa = 10^{10.45}/T^{1.45}$ above 3×10^5 K.

4. Method of Calculation

It is easy to make the calculations given in Tables I and II. We obtained that $\tau_{\rm exp} = 10^{-9}v$, $\tau_{\rm diff} = \kappa\rho^2 C_P R^2/(16\sigma(T^3 - T_S^3))$, $\tau_{\rm cool} = (\tau_{\rm exp}^{-1} + \tau_{\rm diff}^{-1})^{-1}$, $\tau_{\rm rise} = 1.5 \times I0^{10}/v$. We used $\tau_{\rm diff}$ instead of $\tau_{\rm adj}$ since we took into account the fact that when the heated bubble temperature approaches the temperature of its surroundings, its diffusive radiative flux decreases to zero; $dR_+/dt \to 0$ if $T \to T_S$.

We tested these preliminary estimations with detailed numerical calculations, regarding that the material heated by the heat wave of radiative diffusion expanding from the bubble is coupled to it (see Gorbatsky (1964)).

We start by picking up a certain determined virtual value for the radius of the bubble R_V and for its initial virtual temperature surplus $n = T_V/T_S$. At first, in our

TABLE I

The expansion, diffusion, cooling and rising time scales $\tau_{\rm exp}, \tau_{\rm diff}, \tau_{\rm cool}$ and $\tau_{\rm rise}$ in seconds at $t = 0$ for bubbles with different sizes and relative initial temperature surpluses. $T_S = 1.318 \times 10^7$ K is the temperature of the surroundings at $r = 0.1 R_\odot$ from where the bubble starts rising in our calculations. The case with $R_0 = 10^5$ cm.

T_0/T_S	$\tau_{\rm exp}$	$\tau_{\rm diff}$	$\tau_{\rm cool}$	$\tau_{\rm rise}$
1.0001	3.5×10^7	2.6×10^8	3.1×10^7	1.5×10^7
1.001	6.5×10^6	8.9×10^6	3.7×10^6	2.7×10^6
1.01	2.0×10^6	8.3×10^5	5.9×10^5	8.2×10^5
1.1	6.5×10^5	8.0×10^4	7.1×10^4	2.7×10^5

TABLE II

The expansion, diffusion, cooling and rising time scales $\tau_{\rm exp}, \tau_{\rm diff}, \tau_{\rm cool}$ and $\tau_{\rm rise}$ in seconds at $t = 0$ for bubbles with different sizes and relative initial temperature surpluses. $T_S = 1.318 \times 10^7$ K is the temperature of the surroundings at $r = 0.1 R_\odot$ from where the bubble starts rising in our calculations. The case with $R_0 = 10^6$ cm.

T_0/T_S	$\tau_{\rm exp}$	$\tau_{\rm diff}$	$\tau_{\rm cool}$	$\tau_{\rm rise}$
1.0001	1.1×10^7	2.6×10^{10}	1.1×10^7	4.5×10^6
1.001	2.1×10^6	8.9×10^8	2.0×10^6	8.3×10^5
1.01	6.3×10^5	8.3×10^7	6.3×10^5	2.6×10^5
1.1	2.1×10^5	8.0×10^6	2.0×10^5	8.3×10^4

calculations we considered that the bubbles start their rise from $r = R_\odot/10$. Here the mean molecular weight by the standard solar model is $\mu_0 \approx 0.7$. At the very first phase of the bubble formation the density of the heated bubble is $\rho_V = \rho_S$, and $T_v = nT_S$, $Q_V = (2/\mu_0)\pi R_V^3 R_g \rho_V T_V$, $p_V = p_0 = np_S$, $n > 1$. Then we determine the parameters of the bubble which is already in pressure equilibrium with its environment (denoted with indices "0"), $\rho_0 = \rho_V n^{-3/5}$, $R_0 = R_V n^{-1/5}$, $Q_0 = Q_V n^{-2/5}$, $T_0 = Q_0\mu_0/2\pi R_0^3 R_g \rho_0$, $m = \rho_0 V$, and $m_+(t = 0) = 0$ (here m and m_+ are the initial mass of the heated bubble, and the mass of the volume heated by radiative diffusion of the bubble, respectively). Now we pick up a certain set of time steps, and determine the values of the parameters in the next time step using the equations

$$dQ/dt = (dQ/dt)_{ad} + (dQ/dt)_+, \tag{16}$$

where the indices "ad" and " + " refer to the parameters due to adiabatic expansion of the bubble, and due to the addition of new material to the bubble through the heating effect of the radiative diffusion, respectively.

$$dQ_{ad}/dt = 2/5\, Q/p_S (dp/dr) v, \tag{17}$$

where we determine the value of v from Equation (4) .

$$dQ_+/dt = 3/(2\mu) R_g T_S dm_+/dt, \tag{18}$$

$$dm_+/dt = 4\pi \rho_S R^2 dR_+/dt, \tag{19}$$

$$dR_+/dt = 4ac\left(T^3 - T_S^3\right)\big/\left(C_P \kappa \rho_S^2 R\right), \tag{20}$$

$$dR/dt = -1/5 R/p_S (dp/dr) v + dR_+/dt, \tag{21}$$

$$T = ((Q + Q_+)\mu/(1.5 R_g (m + m_+)). \tag{22}$$

We worked with a fourth order Runge – Kutta method to solve the differential equation system. Our calculations differ from the previous ones (Rosenbluth and Bahcall, 1973; Paterno et al., 1997), who worked with $\Delta\rho = 0$, and $\Delta(T/\mu) = 0$ since they assumed homologous, strictly nonradial perturbations. In our calculations, we allowed non-homologous local perturbations, without a strict local hydrostatic equilibrium, and so the heated regions may have initially pressure surplus, too. After a transient period lasting for a few seconds pressure equilibrium sets up, and the bubbles are hotter and less dense than their surroundings, $\Delta p \propto \Delta(T\rho) = 0$, but $\Delta T \neq 0$ and $\Delta\rho \neq 0$.

We solved the differential equation system with a numerical code developed by one of us (G.A.) and generalized by the other author (A.G.). We neglected the radiation pressure in all the terms except the diffusive one. Our method works well below 10^8 K, the estimated errors in each quantities are smaller than 15%.

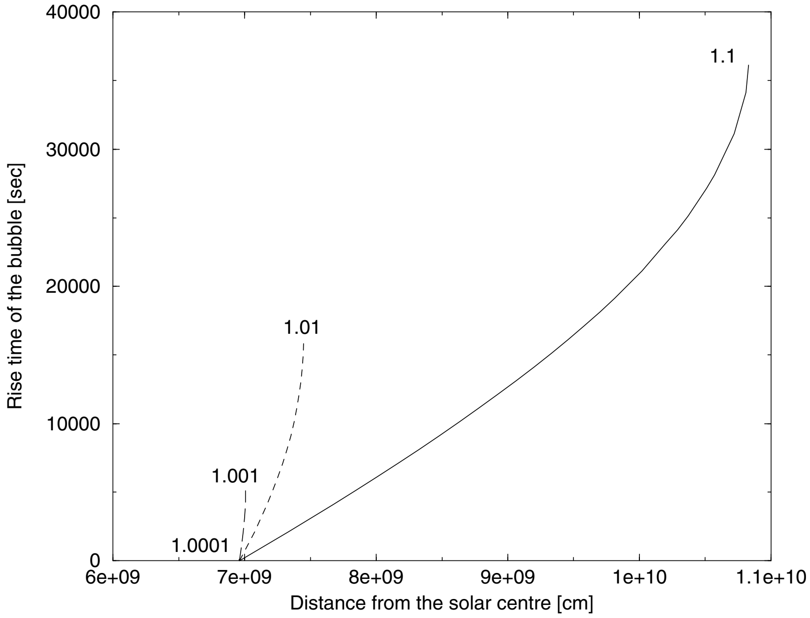

Figure 2. The travel distance of bubbles with different relative initial temperature surplus, for $R_0 = 10^6$ cm. $T_V/T_S = 1.0001, 1.001, 1.01, 1.1$.

5. The Results Of Calculations

Tables I and II show some simple initial results for the moment of the onset of the bubble motion, with the bubble radius at the site of its generation is $R_0 = 10^5$ and 10^6 cm, respectively, with relative initial temperature surplus $\Delta T_0/T_S = 10^{-4}$, 10^{-3}, 10^{-2}, and 10^{-1}. Our calculations indicated that the bubble with $R_0 = 10^5$ cm starts to rise from $0.1R_\odot$ when its heating reaches a temperature surplus above $\Delta T_0/T_S = 7 \times 10^{-5}$, and its energy surplus is $\Delta Q = 6 \times 10^{26}$ ergs.

These figures show us that the characteristic rise time of the bubbles are comparable (or shorter) than the combined cooling time scales, therefore the bubbles are able to rise significant distances in the radiative core. Even when $\tau_{\mathrm{rise}}(t = 0) > \tau_{\mathrm{cool}}(t = 0)$, $\tau_{\mathrm{cool}}(t = 0) \approx 10^5$ sec (Table I, third and fourth rows), and so the bubble already for very small velocities $v \approx 10^3 - 10^4$ cm s^{-1} may travel a distance $\Delta r \approx \tau_{\mathrm{cool}} v \approx 10^8 - 10^9$ cm. For small bubbles (with $R_0 = 10^5$cm) radiative diffusion becomes the dominating cooling agent above $T_0/T_S = 1.001$. For larger bubbles (with $R_0 = 10^6$cm), adiabatic expansion dominates cooling in the calculated range of T_0/T_S.

Our more detailed results, extending now behind the initial state of the bubble, when we followed the rise of the bubbles in time, are shown in Figures 1 and 2. One may observe that already at $\Delta T_0/T_0 = 10^{-4}$ the Δr distance traveled by the bubble with $R_0 = 10^5$ cm is around 1.5×10^6 cm, and for $\Delta T_0/T_0 = 10^{-3}$, Δr is already around 4.8×10^7 cm. For $R_0 = 10^6$ cm, with $\Delta T_0/T_S = 0.1$, the running distance the bubble made is $\Delta r = 3.8 \times 10^9$ cm.

It was Gorbatsky (1964) who considered that a hot bubble generated by a point explosion of unspecified origin may or may not reach the outer region of a star. Unfortunately, his results did not become established. The main reasons could be the apparent implausibility of point explosions in stellar cores and the implausibly enormous energy $Q_0 > 10^{35}$ ergs he needed for a bubble to reach the outer regions of Castor C. His results indicated –by our calculations, erroneously – that hot bubbles may not be important in a star like our Sun. In comparison, our aim was to determine whether the solar radiative interior is metastable or not for finite amplitude perturbations with much lower $Q_0 \approx 10^{20}$–10^{26} ergs. The positive result we obtained is enhanced by two important improvements. These are (i) the improved solar opacities of Rogers and Igliesas (1998), and (ii) the radiative opacity, instead of depending on the 7.5th power of the temperature, as Gorbatsky (1964) calculated, is calculated by the formula of Kippenhahn and Weigert (1990), with a dependence on 3rd power only. Importantly, in this paper we specified some of the mechanisms that may contribute to the necessary perturbations producing the bubbles. Our method approaches the problem from a point different from Gorbatsky's paper, and though we used some important parts of his developments, and confirmed his main idea, we developed a new context and reached substantially new results.

6. Discussion

The obtained results show that there exist two, yet unexplored types of stellar instability within the solar core and similar stellar radiative interiors. When the heating events have amplitudes below a certain energy threshold, which is still below $\approx 10^{27}$ ergs, corresponding to $\Delta T_0/T_S < 1.0001$, our Tables I and II show that they will not form rising bubbles, but heat waves and decay on a diffusion time scale longer than 10^3 years for heating events with spatial scales around $R \approx 10^6$ cm, and this time scale increases to $> 10^7$ years as the generated perturbations become smoothed out and become more extended in space. They modify the solar temperature locally, and this effect may be significant if these small amplitude perturbations are produced frequently enough. Moreover, these perturbations have very long lifetimes and therefore they may overlap each other. This overlapping may be regarded as a new mode of growth of perturbations, which may lead to a new type of instability, when the overlapping effect and the arising increase of temperature is more pronounced than cooling. This process may lead also to generation of heated bubbles that may travel a significant distance. It is not excluded that even very small amplitude heating events (with $\Delta Q \ll 10^{27}$ ergs) may contribute to bubble generation, e.g., when suitably more such perturbations overlap.

Moreover, when perturbations above a certain energy threshold are present, they can directly initiate from time to time large-amplitude individual motions of heated bubbles that can travel a significant length within the solar body. Therefore, they can serve as a new tool with which the solar body may accommodate to quickly changing conditions. If such heated bubbles are present in the solar core, they can generate local, sporadic, "very slow" mixing.

Our results call attention to the principal possibility that local metainstabilities like generation of heated bubbles may explain the rigid rotation of some activity centres (Spence et al., 1993), the existence of sunspot nests (Castenmiller et al., 1986; De Toma et al., 2000), hot spots (Bai et al., 1995), active longitudes (Bai et al., 1995; Bai, 2002, 2003), and, importantly, even the generation of surface activity phenomena like flares.

The importance of our findings is indicated at such far-reaching applications as physics of the Earth, planetary interiors, stellar interiors, and spotted stars.

Interestingly, hot spots occur at the Earth as well, they also show deep origin from the core (Morgan, 1971), rigid rotation (Jurdy and Gordon, 1984), chemical anomalies (Hoffmann, 1997) and surface activity. Recently, using orbit-fixed coordinate systems it became known that a relation exists between the mutual position of the stars of spotted stellar systems and the position of their spots. The phenomenon of active longitudes is known in some close binaries where the position of the two diametrically opposing active longitudes is directly related to the position of the companion star (Olah et al., 2002; Olah, 2002). There is evidence for a purely tidal effect as well-enhanced activity at the subbinary point of close binaries (Cuntz, et al., 2000). Tidal response calculations are also of interest in connection with the newly discovered planets (Terquem et al., 1998), which may play a role in the generation of the activity cycle of their parent stars. A significant positive correlation exists between the chemically 'anomalous' activity of Jupiter's hot spots and the solar cycle (Kostiuk et al., 2000). Moreover, Coraddu et al. (2002a) had shown that moderate deviations from the Maxwell–Boltzmann energy distribution can increase deuterium reaction rates enough to contribute to the heating of Jupiter. These observations seem to point out that the dynamism of solar radiative interior may be coupled not only to rotation and magnetic fields, but also to tidal effects, and, possibly, to solar barycentric motion. The locally unbalanced energy production of the solar core shown in this paper may have consequences in a wide range of solar and stellar physics.

7. Conclusion and Possible Relations to Observations

We demonstrated here for the first time with detailed numerical calculations that finite amplitude nonhomologous perturbations may generate heat waves and local instabilities in the solar core. Our considerations indicated that the presence of these heat waves and heated bubbles may be so significant that they may play an important role in the dynamics of the solar radiative interior. It is shown that when the relative heating of the bubble reaches a certain, low value relative to the available energies, the bubble may travel distances longer than its linear size. Moreover, the small amplitude heat waves represent such deviations from the standard solar models that they may have important effects in the solar behavior.

Our main conclusions are the followings:

- Small amplitude heat waves have a decay time scale that is very long, for $R_0 > 10^6$ cm we obtained $\tau_{\rm decay} > 10^3$ years (see $\tau_{\rm diff}$ in the first row of Table II), and it grows even above the decay time of Paterno, et al. (1997) $\tau_{\rm decay} \approx 10^7$ years as the nonhomologous perturbations smoothes out towards a homologous one. This means that stellar and solar radiative interiors may be in a state of continuous small amplitude oscillation of expanding and contracting heated regions, instead of the quiescent state normally assumed, when significant perturbations are generated frequently enough, and we presented fundamental theoretical and observational reasons showing that such heating events are inevitably setting up in the solar core. The obtained results show that stellar and solar radiative interiors may be rich in subtle dynamical processes, since it may deviate slightly, sporadically and locally, but significantly from local thermodynamic equilibrium. These newly explored effects are the more pronounced since – due to the very long lifetime of perturbations – deviancies from standard structure may accumulate in time, similarly to gravitational resonance effects.
- The small amplitude nonhomologous perturbations and related rotational events may produce local heatings, and these heatings will be fed by the nuclear reactions. Long lifetime microinstabilities produce deviancies from the maxwellian energy distribution. The appearance of nonmaxwellian character of the energy distribution may have observational consequences, see e.g., Coraddu et al. (2002a, 2002b) and Mathai and Haubold (2002). Turck-Chieze (2001) has shown that such effect is not yet excluded in the innermost solar core.
- The appearance of hot bubbles in the solar core may provide a certain dy namism to the solar radiative interior. The dynamic nature of the solar core (Grandpierre, 1996; Turck-Chieze, 2001) is indicated not only by the lithium problem (Deliyannis et al., 1998; Zahn, 2001) and related problems with a need of a kind of mixing in the radiative interior, but also by the anomalously slow rotation of the core, and such anomalous, possibly core-related phenomena as highly localized 10% density fluctuations in the core (Burgess et al., 2003a, 2003b), anomalous heating events (Chaplin et al., 2003), and anomalous rotation rate of anomalously long-living hot-spot systems (Bai, 2003).
- Revealing the presence of metastabilities in the solar core may help our understanding of the different types of instabilities, angular momentum dissipation, spin-down of the solar core and the dynamism arising from its plasma nature.
- Metainstabilities may be related to the generation of the solar (and stellar) activity cycles.
- The obtained results strongly suggest that the Sun deviates significantly from being a gravitationally stabilized quiescent fusion reactor. Indeed, the

Sun appears as a highly complex self-organizing plasma system with a rich dynamism on many time scales simultaneously, and so it may react sensitively to its surroundings.

Acknowledgements

We wish to express our thanks to Dr. Mdzinarishvili for the enlighting discussions, to Prof. Haubold for his continuous interest and encouragement, to Dr. Cserepes for the useful advices on the terrestrial hot spots, and to the anonymous referee for his useful remarks.

References

Allen, C.W.: 1963, *Astrophysical Quantities (3rd edn)*, University of London, The Athlone Press, p. 161.

Bahcall, J.N.: 1989, *Neutrino Astrophysics*, Cambridge University Press, p. 43.

Bai, T., Hoeksema, J.T. and Scherrer, P.H.: 1995, in: J.T. Hoeksema, V. Domingo, B. Fleck and B. Battrick (eds), *Helioseismology* ESA SP, p. 113.

Bai, T.A.: 2002, AAS Meeting 200, No. 91.04.

Bai, T.: 2003, *ApJ* **585**, 1114.

Benevolenskaya, E.E.: 1999, in: SOHO-9 Workshop "Helioseismic Diagnostics of Solar Convection and Activity", Stanford, California, July 12–15, 1999.

Burgess, C., Dzhalilov, N.S., Maltoni, M., Rashba, T.I., Semikoz, V.B., Tortola, M.A. and Valle, J.W.F.: 2003a, *ApJ* **588**, L65.

Burgess, C.P., Dzhalilov, N.S., Rashba, T.L., Semikoz, V.B. and Valle, J.W.F.: 2003b, IFIC-03-12; McGill-02/40; astro-ph/0304462.

Burgess, C., Dzhalilov, N.S., Maltoni, M., Rashba, T.L., Semikoz, V.B., Tortola, M.A. and Valle, J.W.F.: 2003c, astro-ph/0310366.

Castenmiller, M.J., Zwaan, C. and van der Zalm, E.B.J.: 1986, *Sol. Phys.* **105**, 237.

Chang, T., Tarn, S.W.Y., Wu, C.-C. and Consolini, G.: 2003, *Space Sci. Rev.* **107**, 425.

Chaplin, W.J., Elsworth, Y., Isaak, G.R., Miller, B.A., New, R. and Toutain, T.: 2003, *ApJ* **582**, L115.

Charbonneau, P. and MacGregor, B.: 1992, *ApJ* **397**, L63.

Charbonneau, P. and MacGregor, B.: 1993, *ApJ* **403**, L87.

Contopoulos, G.: 2003, in: G. Contopoulos, N. Voglis (eds), *Galaxies and Chaos*, Springer, Berlin Heidelberg New York, p. 3.

Coraddu, M., Lissia, M., Mezzorani, G. and Quarati, P.: 2002a, *Physica* **A305**, 282, http://xxx.lanl.gov/abs/hep-ph/0212054.

Coraddu, M., Lissia, M., Mezzorani, G. and Quarati, P.: 2002b, INFNCA-TH0206, http://xxx.lanl.gov/abs/physics/0112018.

Couvidat, S., Turck-Chieze, S., and Kosovichev, A.G.: 2002, ArXiv.org: astro-ph/0203107.

Couvidat, S., Garcia, R.A., Turck-Chieze, S., Corbard, T., Henney, C.J. and Jimenez-Reyes, S.: 2003, *ApJ* **597**, L77.

Cuntz, M., Saar, S.H. and Musielak, Z.E.: 2000, *ApJ* **533**, L151.

De Toma, G., White, O.R. and Harvey, K.L.: 2000, *ApJ* **529**, 1101.

Deliyannis, C.P., Boesgaard, A.M., Stephens, A., King, J.R., Vogt, S.S. and Keane, M.J.: 1998, *ApJ* **498**, L147.

Dzhalilov, N.S., Staude, J. and Oraevsky, V.N.: 2002, *A&A* **384**, 282.

Gilliland, R.L.: 1985, *ApJ* **290**, 344.
Gorbatsky, L.V.: 1964, *AZh* **41**, 53 (in Russian; English translation in Soviet Astronomy-AJ, 1964, 8, 38).
Grandpierre, A.: 1990, *Sol. Phys.* **128**, 3.
Grandpierre, A.: 1996, *A&A* **308**, 199.
Haber, D.A., Toomre, J., Hill, F. and Gough, D.O.: 1988, Seismology of the Sun and Sun-like Stars, Proc. Symp., Tenerife, Spain, 26–30 September 1988, ESA SP-286 (December 1988), p. 301.
Hoffmann, A.W.: 1997, *Nature* **385**, 219.
Juckett, D.: 2003, *A&A* **399**, 73.
Jurdy, R.M. and Gordon, R.G.: 1984, *J. Geophys. Res.* **89**, 9927.
Kippenhahn, R. and Weigert, A.: 1990, Stellar Structure and Evolution, Springer Verlag, 44.
Kosovichev, A.G. and Zharkova, V.V.: 1998, *Nature* **393**, 317.
Kostiuk, T., Livengood, T., Fast, K., Hewagama, T., Buhl, D., Schmuelling, F. and Goldstein, J.: 2000, *Amer. Astron. Soc.* DPS Meeting No. 32, No. 15.01.
Li, L.H. and Zhang, H.Q.: 1996, *J. Phys. D: Appl. Phys.* **29**, 2217; astro-ph/9711002.
Mathai, A.M. and Haubold, H.J.: 2002, *Ap&SS* **282**, 265.
Morgan, W.J.: 1971, *Nature* **230**, 42.
Olah, K., Strassmeier, K.G. and Granzer, Th.: 2002a, *Astron. Nachr.* **323**, 453.
Olah, K., Strassmeier, K.G. and Weber, M.: 2002b, *A&A* **389**, 202.
Öpik, E.: 1958, *Irish Astron. J.* **5**, 36.
Öpik, E. 1972, *Irish Astron. J.* **10**, 298.
Paterno, L., Rapisarda, L. and Di Mauro, M.P.: 1997, *A&A* **322**, 340.
Peratt, A.L.: 1992, *Physics of the Plasma Universe*, Springer-Verlag, New York-Berlin, 19.
Peratt, A.L.: 1996, *Ap&SS* **244**, 89.
Rogers, F.J. and Igliesas, C.A.: 1998, *Space Sci. Rev.* **85**, 61.
Sornette, D.: 2002, *Proc. of the Nat. Acad. of Sci.* U.S.A. **99**, 2522.
Rosenbluth, M.N. and Bahcall, J.N.: 1973, *ApJ* **184**, 9.
Schwarzschild, M. and Harm, R.: 1973, *ApJ* **184**, 5.
Spence, P., Walker, E.N., Halls, B. and Robertson, L.: 1993, *J. Br. Astron. Assoc.* **103**, 3.
Spiegel, E.A. and Zahn, J.-P.: 1992, *A&A* **265**, 106.
Spruit, H.C. and Knobloch, E.: 1984, *A&A* **132**, 89.
Spruit, H.C.: 2002, *A&A* **381**, 923.
Tayler, R.J.: 1973, *MNRAS* **161**, 365.
Terquem, C., Papaloizou, J.C.B., Nelson, R.P. and Liu, D.N.C.: 1998, *ApJ* **502**, 788.
Toutain, T. and Kosovichev, A.G.: 2001, in *Helio- and Asteroseismology at the Dawn of the Milleneum. Proc. SOHO 10/GONG 2000 Workshop*, Santa Cruz de Tenerife, Spain, 2–6 October 2000 (ESA SP-464, January 2001), 123.
Tsytovich, V.N.: 1970, *Nonlinear Effects in Plasma*, Plenum Press, New York-London, p. 1.
Turck-Chieze, S., Nghiem, P., Couvidat, S. and Turcotte, S.: 2001a, *Sol. Phys.* **200**, 323.
Turck-Chieze, S.: 2001, in: Paul Murdin (ed.), *Encyclopedia of Astronomy and Astrophysics*, Vol. 3, IOP Publ. Bristol, 2616.
Turck-Chieze, S., Couvidat, S., Kosovichev, A.G., Gabriel, A.H., Berthomieu, G., Brun, A.S., Christensen-Dalsgaard, J., Garcia, R.A., Gough, D.O., Provost, J., Roca-Cortes, T., Roxburgh, I.W. and Ulrich, R.K.: 2001, *ApJ* **555**, L69.
Unno, W.: 1975, *PASJ* **27**, 81.
Wolff, C.L.: 2002, *ApJ* **580**, L181.
Zahn, J.-P.: 2001, in: Paul Murdin (ed.), *Encyclopedia of Astronomy and Astrophysics*, Vol. 3, IOP Publ. Bristol, 2563.